CALCULATIONS

Introduction

The overall aim when teaching calculation is to ensure that children understand what each operation actually means. The teaching of calculations should not just give the child a process that produces the correct answer but should equip them with the understanding of what the answer means in real terms. It is infinitely more important that children know why they are doing something as well as how to do it.

The four operations that are covered by this booklet are addition, subtraction, multiplication and division. Whichever operation is being taught the child needs to experience all of these steps to conquer it.

1) using objects
2) using pictures
3) using a numberline
4) using an expanded method
5) using a compact written method

Mental methods

The ability to calculate mentally forms the basis of all methods of calculation. Children should always be encouraged to consider if a mental calculation would be appropriate before using written methods – these are covered in the first part of each section.

Overall Aims

The overall aim is that when children leave primary school they:

- Have a secure knowledge of number facts and a good understanding of the four operations
- Are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to cases involving bigger numbers
- Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads.
- Have an efficient, reliable, compact written method of calculation for each operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally.

GLOSSARY

2-digit – a number with 2 digits like 23, 45, 12 or 60

3-digit – a number with 3 digits like 123, 542, 903 or 561

Addition facts – knowing that 1+1 = 2 and 1+3 = 4 and 2+5 = 7. Normally we only talk about number facts with totals of 20 and under.

Array -An array is an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.

Bridge to ten – a strategy when using numberlines. Adding a number that takes you to the next 'tens' number.

Bus Stop Method - traditional method for division with a single digit divisor

Concrete apparatus – objects to help children count – these are most often cubes (multilink) but can be anything they can hold and move. Dienes (purple hundreds, tens and units blocks), Numicon, Cuisenaire rods are also referred to as concrete apparatus.

Decimal number – a number with a decimal point

Divisor – the smaller number in a division calculation.

Double – multiply a number by 2

Exchanging – Moving a 'ten' or a 'hundred' from its column into the next column and splitting it up into ten 'ones' (or 'units') or ten 'tens' and putting it into a different column

Expanded Multiplication – a method for multiplication where each stage is written, then added up at the end in a column

Find the difference – A method for subtraction involving counting up from the smaller to the larger number

Grid method – a method for multiplying two numbers together involving partitioning

Halve – divide a number by 2

Integer - a number with no decimal point

Inverse – the opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division

Long Multiplication – column multiplication where only the significant figures are noted

Number bonds to ten – 2 numbers that add together to make ten, like 2 and 8, or 6 and 4.

Number bonds to 100 – 2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88

Numberline – a line either with numbers or without (a blank numberline). Children use this tool to help them count on for addition of subtraction and also in multiplication and divison.

Numberline Chunking - method of division involving taking chunks orgroups or the divisor away from the larger number

Number sentence – writing out a calculation with just the numbers in a line E.G. 2+4=6 or 35 ÷7 = 5 or 12 x 3 =36

Partition – split up a larger number into the hundreds, tens and units. E.G. 342 – 300 and 40 and 2

Place Value – knowing that in the number 342 – the '3' means '3 hundreds', the '4' means '4 tens' and the '2' means '2'.

Recombine – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add then hundreds together, then add the tens to that total, then add the units to that total.

Remainder – a whole number left over after a division calculation

Repeated addition – repeatedly adding groups of the same size for multiplication

Significant digit – the digit in a number with the largest value.
E.G in 34 – the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'

Single digit – a number with only one digit. These are always less than 10.

Taking away – a method for subtraction involving counting backwards from the larger to the smaller number

Tens number - a number in the ten times tables – 10,20,30,40 50,etc.

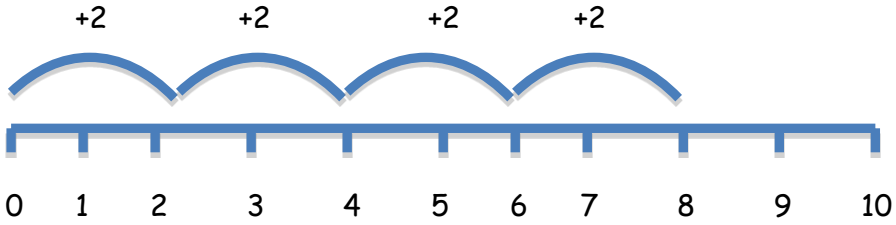
Unit – another term for single digit numbers. The right hand column in column methods is the 'units' column

ADDITION

Ideas and strategies that children should master before tackling written calculations.

These steps are not necessarily taught in order, they will be taught as the child becomes ready.

- Addition can be done in any order. e.g. 34 + 56 = 90 or 56 + 34 = 90
- Usually start with the largest number (so that you have to do less counting and so there is less potential for mistakes.) e.g. 27 + 5 = 32
- Must know number bonds to 10 e.g. 1+9=10, 2+8= 10, 3+7=10, 4+6=10, 5+5=10 etc.
- Addition facts for all single-digit numbers. e.g. 1+1=2, 1+2=3, 1+4=5, 2+1=3, 2+2=4, 2+3=5 etc.
- Count forward in steps of 1, 2, 5, 10 and 100 along a numberline.



- Understand the numberline as a continuum. A numberline is just a tool that helps us count forwards and backwards – it has no 'official' starting or ending point.
- Understand place value. e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7.
- Can partition numbers. e.g. Can split a number like 327 into 300 + 20 + 7
- Counting forwards and backwards in steps of different sizes. e.g. counting forwards in 1s – 1,2,3,4,5 etc; or in steps of 2 – 2,4,6,8,10 etc; or in steps of 5 – 5,10,15,20,25 etc. ; or in steps of 10 – 10,20,30,40,50 etc
- Know doubles of numbers from 1-10 e.g. double 3 is 6, (or 2 lots of 3 is 6, or 2 times 3 is 3, or 2 groups of 3 is 6)
- Know doubles of numbers from 10-20. e.g. double 12 is 24, (or 2 lots of 12 is 24, or 2 times 12 is 24, or 2 groups of 12 is 24)
- Know that adding positive numbers always produces a larger answer.

Addition Vocabulary

+	add	addition	plus
and	count on	more	total
altogether	sum	increase	carrying

ADDITION – PROGRESSION OF METHODS

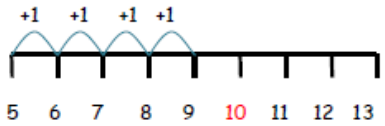
1. Count up to 10 objects reliably.

2. Find 'one more' than a number. e.g. when given a number, say 13, they can count on to find 'one more' e.g. 14.

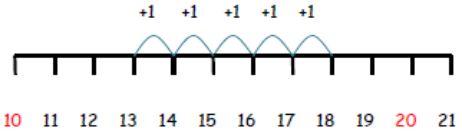
3. Add two or more groups of objects together to find a total of less than 10 using counting on. These may be concrete apparatus or pictures.

4. Use the + and = signs to record mental calculations in a number sentence. eg. 2+4=6

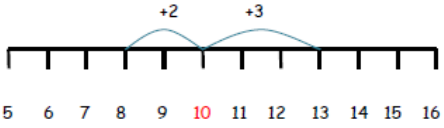
5. Count along a numberline or a number grid to add single digit numbers together to find a total of less than 10 eg. 5 + 4 = 9



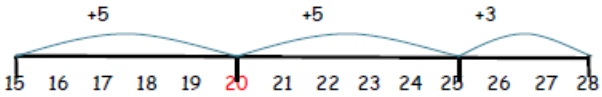
6. Add a 2-digit number and a single digit number using a numberline or number grid e.g. 13 + 5 = 18



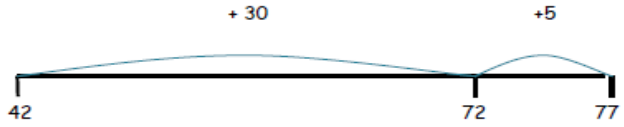
7. Add single digit numbers that bridge to 10 using a numberline or number grid. This involves partitioning the smaller number in to 2 parts, one of which will add to the larger number to make 10 e.g. 8 + 5 = 13



8. Add two 2-digit numbers bridging to 10 using a numberline or number grid. This involves partitioning the smaller number into 2 or more parts, one of which will add to the larger number to make a link to the 'next tens number '
e.g. 13+15 = 28 So split 13 into 5 and 5 and 3.



9. Add two 2-digit numbers adding the most significant digit first using a blank numberline. e.g. 42 + 35= 77



10. Partition and recombine e.g. 15 +13 = 28 (may be done out of order)

15 + 13

10 5 10 3 Then 10 + 10 + 5 + 3 = 28

Standard Written Methods

11. Expanded column addition

84 + 23 = 80 + 4

20 + 3

100 + 7 = 107

12. Standard written column addition without carrying

$$\begin{array}{r} 24 \\ +\ 5 \\ \hline 29 \\ \hline \end{array}$$

13. Standard written column addition with carrying

$$\begin{array}{r} 129 \\ +\ 42 \\ \hline 171 \\ \hline 1 \end{array}$$

14. Adding decimals

$$\begin{array}{r} 126.41 \\ +\ 36.82 \\ \hline 163.23 \\ \hline 11 \end{array}$$

SUBTRACTION

Ideas and strategies that children should master before tackling written calculations.

These steps are not necessarily taught in order, they will be taught as the child becomes ready.

Subtraction can be seen in two ways: as 'taking away' or as 'finding the difference'.

- 'Taking away' is usually used when subtracting a small number from a much larger one – usually 2-digit subtract a single digit like 32 – 6. This is sometimes called 'counting back.
- 'Finding the difference' involves counting up from the smaller to the larger number.
- Must know number bonds to 10 and the reverse. e.g. 1+9=10, 2+8+10, 3+7=10 etc and 10-1=9, 10-2=8, 10-3=7 etc
- Must know number bonds to 100 eg. 20+80 = 100, 45+55=100, 100-43=57, etc
- Understand the numberline as a continuum. A numberline is just a tool that helps us count forwards and backwards – it has no 'official' starting or ending point.
- Subtraction cannot be calculated in any order eg. 9-4=5 is not the same as 4-9 = -5
- Understand place value. eg. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7 units.

Subtraction Vocabulary

-	subtract	take away	minus
less	fewer	difference	decrease
count back	left over	exchange	

SUBTRACTION – PROGRESSION OF METHODS

1. Use concrete apparatus to physically 'take away' from numbers less than 10

2. Find 'one less' than a number. e.g. when given a number, say 13, they can count back to find 'one less' e.g. 12

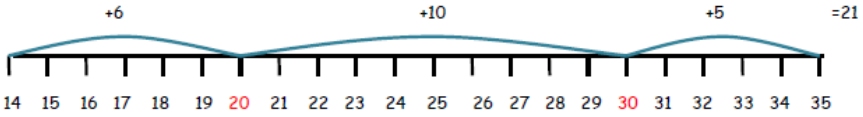
3. Use – and = signs to record mental calculations in number sentences.
   eg. 23- 6 = 17

4. Use concrete apparatus or pictures to either 'take away' or 'find the difference' between 2 groups e.g. 8-3=5

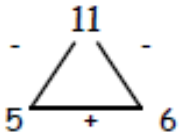
5. Count on/count back in 1s or 10s on a numberline

6. Counting forwards / backwards in steps of different sizes. e.g. counting in 1s, 2s,5,s,10s etc. from any given starting point

7. 'Finding the difference' by counting on. By using a numberline, fingers or other apparatus or mentally count from a smaller number to a larger one.
eg. 35 – 14. Start at 14 and count on to 45. The 'difference' is the answer.

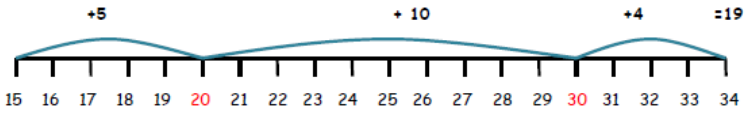


8. Addition/Subtraction inverses (trios)



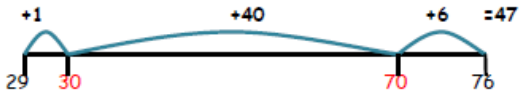
11-6 = 5
11-5 =6
5+6=11

9. Use a numberline for 2-digit numbers subtract 2-digit numbers using 'bridging ten'.
e.g. 34-15 = 19

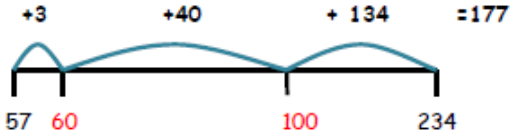
10. 

Use a numberline for

2-digit numbers subtract 2-digit taking bigger jumps to be more efficient e.g. 76 – 29 = 47



11. Use numberline for 3-digit numbers subtract 2-digit numbers using efficient bigger jumps. e.g. 234 – 57 = 177



12. Expanded column subtraction without exchanging

$$\begin{array}{r} 4\ 9\ 9 \\ -\ 3\ 8\ 6 \\ \hline \end{array} = \begin{array}{r} 400 + 90 + 9 \\ -\ 300 + 80 + 6 \\ \hline 100 + 10 + 3 = 113 \end{array}$$

13. Expanded column subtraction with exchanging

$$425 - 143$$

Hundreds	Tens	Units
3~~4~~00	120	5
-100	40	3
200 +	80 +	2 = 282

14. Standard column subtraction without exchanging

T	U
3	5 -
2	4
1	1

15. Standard column subtraction with exchanging

425	3~~4~~25	3~~4~~25
- 143	- 143	- 143
2	82	282

16. Column subtraction for decimals.

$$\begin{array}{r} 5\ \overset{3}{\cancel{4}}.\ \overset{1}{6} \\ -\ 2\ 2.\ 8 \\ \hline 3\ 1.\ 8 \end{array}$$

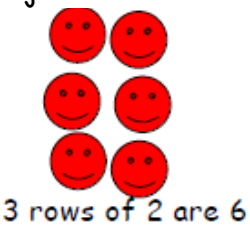
Remember to put the decimal point in your answer space first!

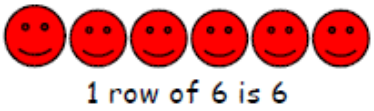
MULTIPLICATION

Ideas and strategies that children should master before tackling written calculations.

These steps are not necessarily taught in order, they will be taught as the child becomes ready.

- Understand place value. e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7
- Recognise simple sequences of numbers. e.g. 5,10,15,20 (add five each time or count in 5s) 2,4,6,8 (add 2 each time or count in 2s)
- Be able to use a method for adding and subtraction (see previous sections)
- Know that multiplication can be calculated in any order e.g. 3x4=12 and 4x3=12
- Be able to show multiplication facts using arrays. You can show a number, e.g. 6, in several ways using pictures or objects





- That multiplication and division are inverse of each other. e.g. 2 x 6 = 12 and 12 ÷ 6 = 2
- Can double and halve numbers from 1 to 100 e.g. Double 4 is 8, 4x2=8; half of 8 is 4, 8÷2=4
- Multiplication is repeated addition. e.g. To find 4 x 3, you add 4 groups of 3, or you add 3 four times: 3+3+3+3 =12

Multiplication Vocabulary

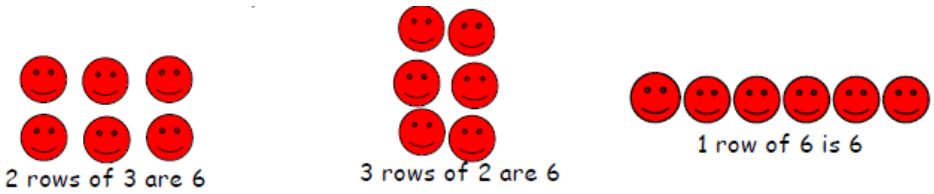
x	lots of	groups of	times
multiply	multiplication	multiple	product
array	double	repeated addition	jump

MULTIPLICATION – PROGRESSION OF METHODS

1. Put objects into groups of the same number

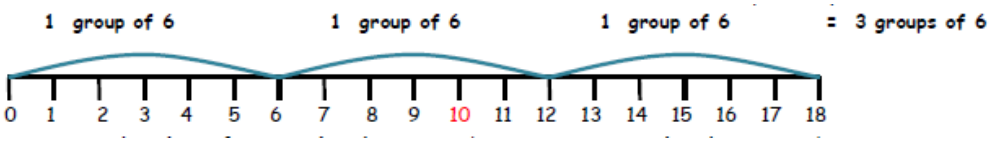
2. Use apparatus to multiply using repeated addition. e.g. 4 x 5 =20 (5 + 5 + 5 + 5)

3. Be able to show multiplication facts using arrays. You can show a number, e.g. 6, in several ways using pictures or objects

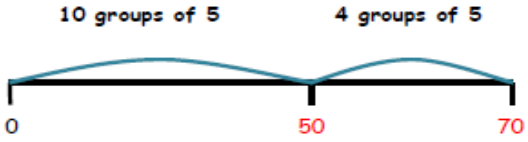


4. Use apparatus alongside a numbered numberline for repeated addition

5. Use a numbered numberline and record the jumps (how many groups of…) for single digit times single digit numbers e.g. 3 x 6



6. Use times tables facts to make more efficient jumps on an open numberline
e.g. for 14 x 5, you could add 10 x 5 and 4x5



7. Use Grid Method for 2-digit numbers times single digit numbers eg. 24 x 3 = 72

1st – Partition 24 into 20 and 4
2nd – Work out 20 x 3 and put the answer in the box
3rd - Work out 4 x 3 and put the answer in the box
4th – Add your answers together either mentally or using column addition

X	20	4
3	60	12

```
  60
+ 12
  72
```

8. Use Grid Method for 2-digit numbers times 2-digit numbers eg. 24 x 32 = 768

1st – Partition 24 into 20 and 4
2nd –Partition 32 into 30 and 2
3rd – Work out 20 x 30 and put the answer in the box
4th – Work out 4 x 30 and put your answer in the box
5th – Work out 20 x 2 and put your answer in the box
6th – Work out 4x2 and put your answer in the box
7th – Add your answers together using column addition

x	20	4
30	600	120
2	40	8

```
  600
+ 120
   40
    8
  768
```

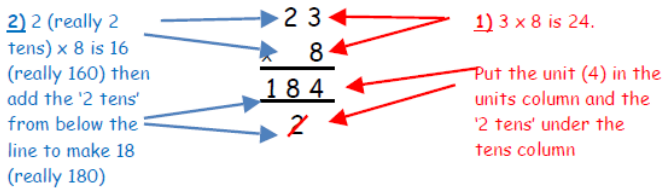
9. Use grid method to multiply decimals eg. 12.5 x 1.6 = 20

1st – Partition 12.5 into 10, 2 and 0.5
2nd – Partition 1.6 into 1.0 and 0.6
3rd – Work out 12 x 1.0 and put the answer in the box
4th – Work out 2 x 1.0 and put your answer in the box
5th – Work out 0.5 x 1.0 and put your answer in the box
6th – Work out 12 x 0.6 and put your answer in the box
7th – Work out 2 x 0.6 and put your answer in the box
8th – Work out 0.5 x 0.6 and put your answer in the box
9th – Add your answers together using column addition

x	10	2	0.5
1.0	10	2	0.5
0.6	6	1.2	0.3

```
   10
    2
+  0.5
    6
    1.2
    0.3
  ------
   20.0
  ------
```

10. Short Multiplication for 2-digit numbers times single digit numbers eg. 23 x 8



11. Expanded Multiplication for 2-digit times 2-digit numbers and beyond eg. 23 x 18

```
  2 3
X 1 8
  2 4  (8x3)
1 6 0  (8x20)
  3 0  (10x3)
2 0 0  (10x20)
-------
4 1 4
-------
 1
```

12. Long Multiplication for 2-digit number times 2-digit numbers eg. 23 x 18

4) 1x3 (really 10x3) is 3 (really 30) – Write the 3 in the tens column.

5) 1x2 (really 10x20) is 2 (really 200) Write the 2 in the hundreds column

6) Add up both of your 'mini answers'

```
  2 3
x 1 8
-------
1 8 4
2 3 0
-------
4 1 4
-------
 1
```

1) 8x3 is 24. Write the 4 in the units column and the 2(really 2tens) under the tens column.

2) 8x2(really 8x20) is 16 (really 160) add the 2 tens from below the line to make 18 (really 180)

3) Place a '0' in the units column as everything will now be multiplied by a 'tens number'.

13. Long multiplication for decimal numbers eg. 23.4 x 64.7

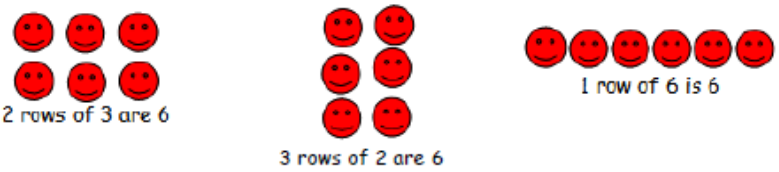
```
  23.4
x 64.7
-------
```

DIVISION

Ideas and strategies that children should master before tackling written calculations.

These steps are not necessarily taught in order, they will be taught as the child becomes ready.

- Understand place value. e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7
- Put objects into groups of the same number.
- Recognise simple sequences of numbers. e.g. 5,10,15,20 (add five each time or count in 5s) 2,4,6,8 (add 2 each time or count in 2s)
- Be able to use a method for adding and subtraction (see previous sections)
- Recall multiplication facts up to 12 x 12 and derive division facts. e.g. 5 x 4 = 20, so 20 ÷ 5 = 4 and 20 ÷ 4 = 5
- Be able to show multiplication facts using arrays. You can show a number, e.g. 6, in several ways using pictures or objects



- That multiplication and division are inverse of each other
  eg. 2 x 6 = 12 and 12 ÷ 6 = 2
- Can find a half and a quarter of a group of objects or a whole number
- Can double and halve numbers from 1 to 100 e.g. Double 4 is 8, 4x2=8; half of 8 is 4, 8÷2=4
- Know that division cannot be calculated in any order e.g. 12÷4=3 is not the same as 12÷3=4
- Can divide by 10 and 100

Division Vocabulary

÷	lots of	groups of	share
group	jumps	equal	halve
divide	division	divided by	remainder
factor	decimal	decimal place	divisible

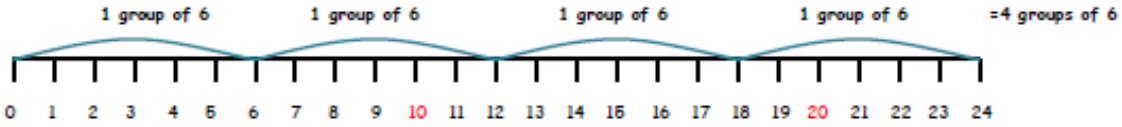
DIVISION – PROGRESSION OF METHODS

1. Share objects into groups of equal size.

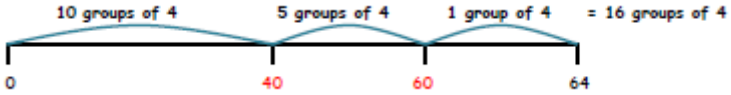
2. Use apparatus to divide numbers into chunks of equal size e.g. 24÷6 = 4

3. Use dots/pictures and circles on paper e.g. 24÷6 =4

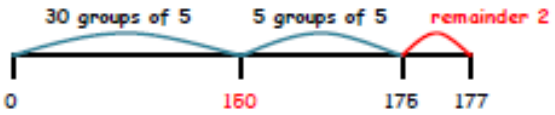
4. Repeated addition using a numbered numberline eg. 24 ÷6 = 4



5. Use Numberline Chunking for 2-digit numbers divided by single digit numbers
eg. 64 ÷4 =16

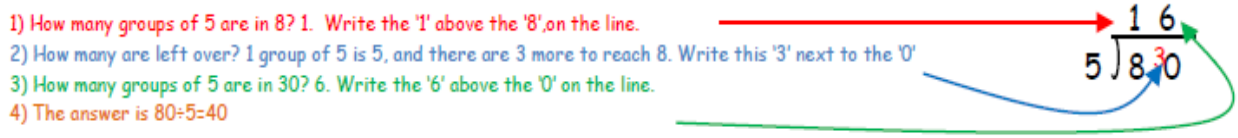


6. Use Numberline Chunking for 3-digit numbers divided by single digit numbers with remainders (using more efficient jumps) e.g. 177÷5



7. Use Numberline Chunking for 3-digit numbers divided by 2-digit numbers with remainders.

8. Use the Bus Stop Method to divide a 2-digit number by a single digit number e.g. 80÷5 =



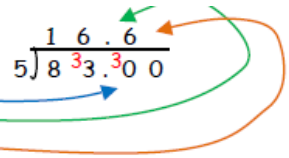
9. Use the Bus Stop Method to divide a 2-digit number by a single digit number with remainders e.g. . 83 ÷5 = 16r3

$$5\overline{)8\,3}\quad 1\,6\ r3$$

10. Use the Bus Stop Method to divide a 3-digit number by a single digit number with remainders e.g. . 483 ÷5

11. Use the Bus Stop Method to divide a 3-digit number by a single digit number with a decimal answer e.g. 83 ÷5 = 16.6

1) Complete the steps until you reach the point where there would be a remainder THEN
2) Put a decimal point and two '0' after the big number
3) Put a decimal point after the last number on the line.
4) How many groups of 5 are in 30? 6. Write the '6' above the line.



12. Use the Bus Stop Method to divide a decimal number by a single digit number with a decimal answer e.g. 83.7 ÷5 = 16.74

13. Use Long 'Bus Stop' Division to divide a 3-digit number by a 2-digit number with a decimal answer e.g. 462÷13 =35.53

1) Set out the numbers for the calculation (divisor on the left) and put in a decimal point and two '0's 13) 4 6 2 . 0 0	2) How many groups of 13 are in 4? None. Write a '0' above the 4. 3) How many Groups of 13 are in 46? 3. Write a '3' above the '6' 0 3 13) 4 6 2 . 0 0
4) What is 3 x 13? 39. Write this '39 underneath the '46' and subtract it. Write the answer '7' underneath the '9' 0 3 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7	5) Bring down the '2' and write it next to the '7' 0 3 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2
6) How many groups of 13 are there in 72? 5. Write the '5' above '2' on the answer line 0 3 5 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2	7) What is 5 x 13? 65. Write '65' below the '72' and subtract it. Write the answer '7' underneath the '5'. 0 3 5 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2 (5x13 = 65) - 6 5 7
8) Put the decimal point into the answer line. 9) Bring down the '0' and write it next to the '7' 0 3 5 . 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2 (5x13 = 65) - 6 5 7 0	10) How many groups of 13 are in 70? 5. Write the '5' on the answer line above the '0' 0 3 5 . 5 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2 (5x13 = 65) - 6 5 7 0

11) What is 5 x13? 65. Write the 65 below the 70 and subtract it. Write the answer 5 underneath the '5'. 0 3 5 . 5 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2 (5x13 = 65) - 6 5 7 0 (5x13=65) - 6 5 5	12) Bring down the next '0' and write it next to the '5' 0 3 5 . 5 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2 (5x13 = 65) - 6 5 7 0 (5x13=65) - 6 5 5 0
13) How many groups of 13 are in 50? 3. Write the '3' above the '0' on the answer line. 0 3 5 . 5 3 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2 (5x13 = 65) - 6 5 7 0 (5x13=65) - 6 5 5 0	14) What is 3 x 13? 39. Write '39' below the '50' and subtract it. Write the answer '11' underneath the '5'. 0 3 5 . 5 3 13) 4 6 2 . 0 0 (3 x13=39) - 3 9 7 2 (5x13 = 65) - 6 5 7 0 (5x13=65) - 6 5 5 0 (3x13=39) - 3 9 1 1
15) Now there are two decimal places in the answer, you can stop working!	